Letter to the Editor: Comments on "A new method of processing capillary viscometry data in the presence of wall slip" [J. Rheol. 47, 337–348 (2003)]

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Mooney's procedure, involving multiple capillary dies with different diameters, is generally used for the wall slip analysis of capillary flow data [Mooney (1931)]. Yeow *et al.* (2003) assert that the method suggested by Mooney constitutes an "ill-posed problem," and furthermore requires the "assumption of a rheological model to relate the shear rate and the slip velocity to the local shear stress." Are these assertions correct?

The velocity distribution, $V_z(r)$ during the fully developed flow of an incompressible fluid under isothermal conditions in a straight tube with radius, R, and subject to a slip velocity of U_s at the wall is

$$V_z(r) = U_s + \int_r^R \dot{\gamma}\, dr, \tag{1}$$

where the shear rate, $\dot{\gamma}$, is $(-dV_z/dr)$. The mechanism of the wall slip and the relationship between the slip velocity, U_s, and the wall shear stress, τ_w, need not be known. The integration of the velocity distribution for the flow rate, Q, followed by integration by parts, substitution of the shear stress, $\tau_{rz} = \tau_w(r/R)$, into the integral, taking the derivative with respect to the wall shear stress, τ_w, and using the Leibniz rule of integration gives the following for the true wall shear rate at a given wall shear stress, i.e., $\dot{\gamma}(\tau_w)$:

$$\dot{\gamma}(\tau_w) = \left(\frac{Q-Q_s}{\pi R^3}\right)\left[3+\frac{d\ln[(Q-Q_s)/\pi R^3]}{d\ln \tau_w}\right], \tag{2}$$

as given in Yilmazer and Kalyon (1991) and Kalyon *et al.* (1993). Here, Q_s is the volumetric flow rate due to slip, i.e., $Q_s = U_s \pi R^2$, where U_s is obtained from [Mooney (1931)]:

$$U_s(\tau_w) = \left.\frac{\partial(Q/\pi R^3)}{\partial(1/R)}\right|_{\tau_w}, \tag{3}$$

For the no-slip condition, i.e., U_s and $Q_s = 0$, the Rabinowitsch equation is reclaimed [Rabinowitsch (1929)]:

$$\dot{\gamma}_w(\tau_w) = \frac{Q}{\pi R^3}\left[3+\frac{d\ln(Q/\pi R^3)}{d\ln \tau_w}\right]. \tag{4}$$

Thus, the solution for the slip-corrected wall shear rate, $\dot{\gamma}_w(\tau_w)$ under conditions in which the flow is not plug flow, i.e., $Q_s/Q \neq 1$, is given as

$$\dot{\gamma}_w = \left(\frac{Q}{\pi R^3} - \frac{1}{R} \left(\frac{\partial (Q/\pi R^3)}{\partial (1/R)} \right) \Bigg|_{\tau_w} \right) \left(3 + \frac{d \ln \left(\frac{Q}{\pi R^3} - \frac{1}{R} \frac{\partial (Q/\pi R^3)}{\partial (1/R)} \right)}{d \ln \tau_w} \right). \quad (5)$$

For plug flow conditions $Q = U_s \pi R^2$ and there may be other complications [Yaras *et al.* (1994)].

Equation (5) indicates that there exists a solution (in fact, a unique solution) for the determination of the true wall shear rate in capillary viscometry under wall slip conditions. Furthermore, there is no need to assume a constitutive equation for the fluid or a specific relationship between the wall shear stress and the slip velocity. Does this solution depend continuously on the data provided and do small changes in the data give rise to equally small changes in the solution? For wall shear stress and flow rate data that are experimentally generated under systematic error conditions and when the slope

$$\left[\frac{d \ln[(Q - Q_s)/\pi R^3]}{d \ln \tau_w} \right]$$

exhibits relatively moderate values, the solution is generally stable. On the other hand, when the experimental random errors are severe, or under conditions where the velocity distribution approaches the plug velocity condition, stability problems can ensue. However, even under such conditions it is prudent to analyze the data first by using this analysis method, instead of solely attempting a numerical solution for the values of the true shear rate at the wall.

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References

Kalyon, D., P. Yaras, B. Aral, and U. Yilmazer, "Rheological behavior of a concentrated suspension: A solid rocket fuel simulant," J. Rheol. **37**, 35–53 (1993).

Mooney, M., "Explicit formulas for slip and fluidity," J. Rheol. **2**, 210–222 (1931).

Rabinowitsch, R., "Über the viskosität and elastizität von solen," Z. Phys. Chem. Abt. A **145**, 1–26 (1929).

Yaras, P., D. Kalyon, and U. Yilmazer, "Flow instabilities in capillary flow of concentrated suspensions," Rheol. Acta **33**, 48–59 (1994).

Yeow, Y. L., H. L. Lee, A. R. Melvani, and G. C. Mifsud, "A new method of processing capillary viscometry data in the presence of wall slip," J. Rheol. **47**, 337–348 (2003).

Yilmazer, U., and D. Kalyon, "Dilatancy of concentrated suspensions with Newtonian matrices," Polym. Compos. **12**, 226–232 (1991).